Inversion of Electrical Conductivity Parameters in Double-Layered Earth with 3-Dimensional Anomalies

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Electromagnetic detection is one of the most important methods of geophysical exploration. It is superior to other methods in remote sensing telemetry and subsurface fluid detection. One of the main purposes of electromagnetic detection is to obtain the distribution of electromagnetic parameters of target objects through electromagnetic imaging. Interpretation and inferences can be made from the distribution of such parameters. The differences of electromagnetic parameters of different objects are the physical basis of electromagnetic imaging, whereas the theoretical basis is the characterization of electromagnetic parameters by medium types.

In electrical prospecting on three-dimensional geoelectric structures, the most important methods employed by numerical simulations include finite difference, finite element and integral equation. The first two methods are to divide the whole space,whereas the latter needs only to divide anomalies, and simulates the responses of anomalies of limited size.

The geoelectricity study we present here targets double-layered earth structures with three-dimensional anomalies. Scattering and superposition methods are used to deduce a two-layer Dyadic Green's function. With BVP transforms of integral equation, we can obtain the required electromagnetic parameters, such as the conductivity values.

The geoelectricity framework is illustrated by Figure 1 below. The earth is divided into two layers: the upper is the air layer, the lower is the soil layer which contains anomalies. Both are considered homogeneous and isotropic. Resistivity can change in the anomalies and the earth is excited by the impressed current. Suppose the permeability of soil μ is μ_0, and we ignore the current displacement.

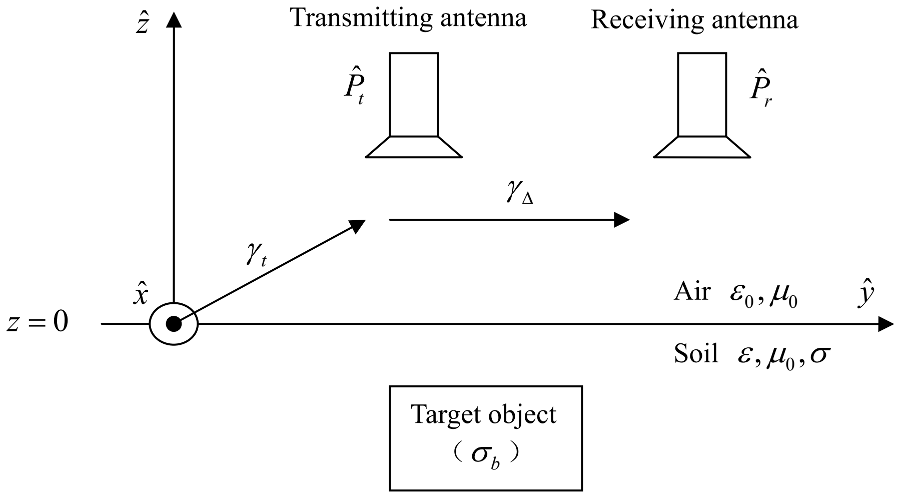

Figure 1

In the frequency domain, Maxwell's equations are as follows:

$$\nabla \times \mathbf{E} = i\omega\mu_0 \mathbf{H} \tag{1}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + (\sigma - i\omega\varepsilon_0)\mathbf{E} \tag{2}$$

Two zones, namely the wave number：$k_1^{\ 2} = \omega^2 \mu_0 \varepsilon_0 \approx 0$，$k_2^{\ 2} = \omega^2 \mu_0 \varepsilon_2 = i\omega\mu_0\sigma_b$。

According to Dyadic Green's function theory, the third category will be used. The solution of electric field strength from (1) and (2) is equivalent to that of the following integral equation:

$$\mathbf{E(R)} = \mathbf{E}_p\mathbf{(R)} + i\omega\mu_0 \int_{V_A} (\sigma - \sigma_b)\bar{\bar{G}}_e\mathbf{(R, R')} \cdot \mathbf{E(R')} dV' \tag{3}$$

This is a second class of singular, vector Fredholm integral equation. Here $\mathbf{E}_p\mathbf{(R)}$ is the primary field of exciting source, and $\bar{\bar{G}}_e\mathbf{(R, R')}$ is the third type of Dyadic Green's function. The electric field in V_A can be derived from the (3). By applying the appropriate Dyadic Green's functions, the electric field at any point of the space can be obtained..

To facilitate numerical calculation, the study area is divided into unit cubes, and the conductivity of the discrete element is set to a constant, that is, each unit cube has a constant electric field (3) can then be rewritten as the following form of discrete units:

$$\mathbf{E}(\mathbf{R}) = \mathbf{E}_p(\mathbf{R}) + i\omega\mu_0 \sum_{n=1}^{N} (\sigma_n - \sigma_b) \int_{V_A} \bar{\bar{G}}_e(\mathbf{R}, \mathbf{R}') dV' \cdot \mathbf{E}_n \tag{4}$$

The focal point of the electric field for the m-th unit cube can be expressed as

$$\mathbf{E}_m = \mathbf{E}_{pm} + i\omega\mu_0 \sum_{n=1}^{N} (\sigma_n - \sigma_b) \int_{V_A} \bar{\bar{G}}_e(\mathbf{R}_m, \mathbf{R}') dV' \cdot \mathbf{E}_n \tag{5}$$

By rewriting the above as a matrix equation, we get

$$\sum_{n=1}^{N} [i\omega\mu_0 (\sigma_n - \sigma_b) \int_{V_A} \bar{\bar{G}}_e(\mathbf{R}_m, \mathbf{R}') dV' - \boldsymbol{\delta}_{mn}] \cdot \mathbf{E}_n = -\mathbf{E}_{pm} \tag{6}$$

where

$$\boldsymbol{\delta}_{mn} = \begin{cases} \mathbf{I} & when \;\; m = n \\ \mathbf{0} & when \;\; m \neq n \end{cases} \qquad m = 1, 2, 3, \cdots, N \tag{7}$$

And $\mathbf{I}$ is the unit vector. By solving (6), we can obtain the focal point for each discrete element of the electric field. The discrete electric field at any point can then be calculated from (4).

Questions:

(**1**) How do we accurately calculate the volume integral of the Green's function of about cube of the current $\int_{V_A} \bar{\bar{G}}_e(\mathbf{R}_m, \mathbf{R}') dV'$?

(**2**) When we consider the frequency of excitation in different conditions, the first focal point of the electric field of each cell can be expressed as:

$$\mathbf{E}_m^{(1)} = \mathbf{E}_{pm}^{(1)} + i\omega_1\mu_0 (\sigma_A - \sigma_b) \sum_{n=1}^{N} \int_{V_A} \mathbf{G}(\mathbf{R}_m, \mathbf{R}') dV' \cdot \mathbf{E}_n^{(1)} \tag{8}$$

$$\mathbf{E}_m^{(2)} = \mathbf{E}_{pm}^{(2)} + i\omega_2\mu_0 (\sigma_A - \sigma_b) \sum_{n=1}^{N} \int_{V_A} \mathbf{G}(\mathbf{R}_m, \mathbf{R}') dV' \cdot \mathbf{E}_n^{(2)} \tag{9}$$

It remains to be studied as how to calculate the conductivity parameters σ_b ?